Ph. D. Ente. test
Statistics.

Con. 3229–12. KK–4376

(3 Hours) [Total Marks : 100

N.B. : 1. All questions are compulsory.
2. Figures to right in the bracket indicate the marks.
3. Calculators are not allowed.

Section I

All questions are compulsory. (20X2 =40 marks)

1) Let A and B be any two events in a probability space such that P(A)=0.6 and P (B)=0.7 which of the following statement is false.

A) $P(A\cup B)=1$ B) $P(A\cap B)=0.6$
C) $P(A\cap B)<0.3$ D) $P(A\cap B)\geq 0.3$

2) X is a non negative integer valued random variable such that
$P[X\geq 1]=15/16$, $P[X\geq 2]=11/16$, $P[X\geq 3]=5/16$, $P[X\geq 4]=1/16$,
$P[X\geq r]=0$ $for\ r>4$ hence

A) The sum of probabilities of values of X is greater than one
B) $P[X=0]=1/8$
C) Mean of X is equal to 2
D) $P[X=5]=1/16$

3) A random sample of size n is drawn from a population of size N by the method of simple random sampling without replacement (SRSWOR). The probability that the i^{th} unit of the population is not included in the sample is

A) $\frac{n-1}{n}$ B) $\frac{N-n}{N}$ C) $\frac{N-1}{N}$ D) $\frac{n-1}{N-1}$

4) Suppose N is not a multiple of n then the number of units selected systematically with sampling interval k(= integer part of $\frac{N}{n}$) is

A) n B) n+1 C) n or n+1 D) k+1

5) For a standard linear model $Y=X\beta+\varepsilon,\quad \varepsilon\sim N(0,\sigma^2 I)$. $\hat{\beta}$ denotes a solution of Normal Equations and $l'\beta$ is estimable. Which of the following is not true?

A) $l'\hat{\beta}$ is Biased B) $l'\hat{\beta}$ is M.L.E. C) $l'\hat{\beta}$ is Unique D) $l'\hat{\beta}$ is Linear

6) For the two way ANOVA model $y_{ijk}=\mu+\alpha_i+\beta_j+e_{ijk}$ i= 1......4 , j=1.....5, k=1,2. the error degrees of freedom are

A) 20 B) 26 C) 12 D) 27

7) Which of the following is not true for $H = X(X'X)^{-1}X^{1}$ matrix in regression ?

A) Idempotent B) symmetric C) orthogonal D) trace (H)=Rank (X)

8) Consider the interval (0,1) with $A = \left\{\left[0, \frac{1}{2}\right]\right\}$ and $B = \left\{\left[\frac{1}{2}, 1\right]\right\}$ then $P(A \cup B)$ is

A) 1 B) 0 C) 0.75 D)0.25

9) Let N be a random variable and $P[N = n] = \frac{1}{n(n+1)}$, n= 1,2,....., ∞, then EN is

A) 2 B) $\frac{7}{8}$ C) $\frac{5}{8}$ D) infinity

10) Let $X_1, X_2, ...X_n$ be iid geometric random variables with parameter p, Then the distribution of $X_{(1)} = \underset{1 \le i \le n}{Min}\ X_i$ is

A) Binomial B) Poisson C) Geometric D)Hypergeometric

11) If the random variable X has poisson distribution with parameter λ and the conditional distribution of y given X=x is binomial with parameters x and p (0<p<1), then the distribution of Y is

A) Binomial with n and p B) poisson with λp

C) geometric with p D) poisson with $\frac{\lambda}{p}$

12) Let a r.v. X have Bernoulli distribution defined by $P[X = 1]$=1- $P[X = 0] = \theta$, where $0 < \theta < 1$. The maximum likelihood estimator of θ based on X is

A) $\frac{1-X}{3}$ B) 1-X C) $\frac{3-X}{3}$ D)X

13) The probability density function of the random variable X follows the following probability density function

$$f(x) = \frac{1}{2\theta}\exp\left[-\frac{|x-\theta|}{\theta}\right], \quad -\infty < x < \infty$$ Mean of this distribution is

A) $\frac{1}{\theta}$ B) θ C) 2θ D)0

14) In 2^4 factorial experiment with two blocks of eight plots each in a replication it was decided to confound ABCD and blocks were constructed as shown below.

Block 1: ab, ac, ad, bc, bd, cd, x_1, x_2

Block 2: abc, abd, acd, bcd, a, b, y_1, y_2

Identify the treatment combination x_1 , x_2 from block 1 and y_1 , y_2 from block 2.

A) $x_1 = abcd$, $x_2 = c$, $y_1 = (1)$, $y_2 = d$

B) $x_1 = c$, $x_2 = d$, $y_1 = (1)$, $y_2 = abcd$

C) $x_1 = abcd$, $x_2 = (1)$, $y_1 = c$, $y_2 = d$

D) $x_1 = (1)$, $x_2 = d$, $y_1 = abcd$, $y_2 = c$

15) Let N be the incidence matrix of a BIBD with parameters v, b, r, k, λ. Which of the following is true?

A) det $(N'N) = 0$

B) b is greater than or equal to v.

C) Any two blocks have λ treatments common.

D) det N is integer.

16) The trace of C matrix in general block design with v treatments replicated $r_1 \le r_2 \ldots\ldots \le r_v$ times and b blocks having $k_1 \le k_2 \ldots\ldots \le k_b$ plots is given by

A) Equal to $b(k_b - 1)$

B) Less than $b(k_b - 1)$

C) Greater than $b(k_b - 1)$

D) equal to $v(r_v - 1)$

17) Determine the values of w, x, y, z such that L_1 and L_2 given below are mutually orthogonal

L_1

0	1	2	3
1	0	3	2
2	3	0	1
3	2	1	0

L_2

0	3	1	2
1	2	0	3
2	1	z	w
3	0	y	x

A) w=0 x=1 y=2 z=3

B) w=3 x=0 y=1 z=2

C) w=2 x=3 y=0 z=1

D) w=1 x=2 y=3 z=0

18) Let X be $N_p(\mu,\Sigma)$ then it is said to be singular normal distribution if and only if

A) $\mu = 0$ B) $\mu \neq 0, |\Sigma| \neq 0$ C) $|\Sigma| = 0$ D) $\mu = 0, |\Sigma| = 0$

19) Let X be $N_3(\mu,\Sigma)$ with $\mu' = (2 \quad -3 \quad 1)$ and $\Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$, then the distribution of $3X_1 - 2X_2 + X_3$ is

A) $N(13,9)$ B) $N(13,3)$ C) $N(-13,3)$ D) $N(-13,9)$

20) Let X be $N_3(\mu,\Sigma)$ with $\mu' = (1 \quad -1 \quad 2)$ and $\Sigma = \begin{pmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{pmatrix}$, Which of the following random variables are not independent?

A) X_1 and X_2 B) X_2 and X_3 C) X_1 and X_3 D) $(X_1 \quad X_3)$ and X_2

Section II

Attempt any three(03) questions out of five (05). (3X10=30 marks)

1) a) Find the characteristic function of Cauchy random variable

b) Explain proportional allocation, optimum allocation and Neyman allocation in stratified sampling; Show that $V_{prop}(\bar{y}_{st}) \geq V_{opt}(\bar{y}_{st})$.

2) a) State the two way random effect model with equal observations per cell (≥ 2) with interaction. State the assumptions. Give the ANOVA table alongwith Expected Mean Sum of squares and obtain estimates of variance components.

b) Let X be distributed as $N_3(\mu,\Sigma)$ where $\mu' = (1 \quad -1 \quad 2)$ and $\Sigma = \begin{pmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{pmatrix}$

Find conditional distribution of X_1 given that $X_3 = x_3$ and conditional distribution of X_1 given that $X_2 = x_2$ and $X_3 = x_3$.

3) a) $X_1, X_2, \ldots\ldots X_n$ are iid rvs with $U(\theta_1, \theta_2)$ find the sufficient statistics for θ_1 and θ_2. Find the conditional distribution of X_1, X_2, X_3 given T, where T is a sufficient statistics for (θ_1, θ_2).

b) Let T_1, T_2 be two unbiased estimators having common variance $\alpha\sigma^2 (\alpha > 1)$ where σ^2 is the variance of UMVUE. Show that the correlation coefficient between T_1 and T_2 is $\geq \frac{2-\alpha}{\alpha}$.

4) a) Let X be a random variable with the following probability density function

$$f(x,\theta) = \frac{2(\theta - x)}{\theta^2} \quad ; \quad 0 < x < \theta$$

$$= \quad 0 \quad ; \quad otherwise$$

Find UMP test for testing

$$H_0 : \theta = \theta_0 \quad \text{against} \quad H_1 : \theta = \theta_1 > \theta_0$$

b) Define

(i) Unbiased test

(ii) Locally most powerful unbiased test

(iii) Neyman-structure test

5) We are interested in the total weight of v objects by means of spring balance. Naturally all the objects cannot be weighted simultaneously. We take the weighing design $X = N'$ where N is the incidence matrix of BIBD with parameters v, b, r, k, λ. Obtain the variance of the best linear unbiased estimate of the total weight.

Section III

Attempt any two (02) questions out of three (03) (2X15=30 marks)

1) a) A discount store records indicate that daily demands for a certain item can be represented by the following distribution.

Demand	0	1	2	3	4	5	6	7	8
Probability	0.1	0.1	0.2	0.1	0.1	0.1	0.1	0.1	0.1

Based on this the storekeeper has set up an inventory system as follows. The store starts the day with at least three items but not more than 7. If during the day the inventory level falls below 3, new items are ordered and the following day starts with a level 7 again. It is assumed that ordered items arrive by the following morning. Orders are not placed for the items if the inventory level does not go below 3. Describe the state space for inventory level at the eginning of the day and show that it is a Markov chain. Determine its transition probability matrix.

b) Define ridge estimator. Obtain its bias. Also obtain bounds for the ridge parameter.

2) a) If X_1 and X_2 are independent $N(0,\sigma^2)$ random variables

i)Find the joint distribution of Y_1 and Y_2 where

$$Y_1 = X_1^2 + X_2^2 \text{ and } Y_2 = \frac{X_1}{\sqrt{Y_1}}$$

ii)Show that Y_1 and Y_2 are independent

b) $X_1, X_2 \ldots . X_n$ are iid random variables with negative binomial distribution with parameters k and p. Find the UMVUE of $p^r q^s$ where $q = 1 - p$.

3) a) Let N be the incidence matrix of a BIBD with parameters v, b, r, k, λ. Show that eigenvalues of $A = (r-\lambda)^{-1}[N'N - (\lambda v/b)J]$ are zero or one. Using this result obtain the bounds for number of treatments common between any two blocks of BIBD.

b) Consider the p.d.f. given below

$$f(x_1, x_2, x_3) = k \quad (x_1 + x_2 x_3) \quad , \quad 0 < x_1, x_2, x_3 < 1$$
$$= 0 \quad , \quad \text{otherwise}$$

i) determine K

ii) Obtain the mean vector

iii) Obtain variance covariance matrix.
